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.Mathematical Modeling on Peristaltic Transport of Two Layered Viscous Incompressible Fluid in Relation to Varying Wall Moment

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Abstract

The present paper concerns study of peristaltic transport in relation to temperature effect and varying wall moment. A two layered peristaltic fluid flow model is employed in estimating the physiological flow parameters (velocity, flux, limitation of flux). Peristaltic flows are assumed to be the propagation waves along the flexible walls which can be compared with the transport of blood within the small blood vessels. The elasticity of varying wall consists of distensibility of the layer. The wall of the tube in a man is accounted for four coats and three neural mechanisms. The coats namely serous, muscular submucus and mucus play a role in the forward propulsion and the excellent lubricant. Therefore a series of contractions of the progressive waves enable the fluid to be transported under the peristaltic action. The resulting wave is sinusoidal due to the longitudinal and transverse moments produced by muscular fibers. The amplitude of the traveling wave on the elastic wall is so large that at the narrowest point the wall is pressed by each other. Numerical method is employed for the analytical expression as series form and the corresponding flow rate is studied in relation to peripheral circulation.

Keywords:

Peristaltic;
blood;
wave;
flow;
flux;
Herschel-Bulkley fluid.

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1. INTRODUCTION

The artery wall consists of three coats namely intima, media and advantia. The inner wall intima of the artery includes the delicate lining endothelium on which the peristaltic effect approximates the flow impact on variable wall moment. Normally intestine, ureter, movement of spermatozoa and the number of biomedical instruments such as some heart lung machines have been identified for the propelling action of the fluid and the fluid mixtures. By the unique pumping process, the fluid is transported by regular coordinated waves of muscular contractions along the wall of the vessel. The vagus nerve functions as a fine

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tuner to stimulate the peristaltic moments. The chemical and mechanical stimulations of peristaltic wave are well supported for its movement by the gel(mucus). This is alkaline secreted by columnar cells serves as a protective function. The gel prevents the damage of the wall along the progressive wave. The wall of the tube in a man is accounted for four coats and three neural mechanisms. The coats namely serous, muscular submucus and mucus play a role in the forward propulsion and the excellent lubricant. Therefore a series of contractions of the progressive waves enable the fluid to be transported under the peristaltic action. The resulting wave is sinusoidal due to the longitudinal and transverse moments produced by muscular fibers. Human aorta, carotid artery, capillaries and other vessels of the sizes constituted by connective tissues followed by smooth muscular fibers are contractile in nature. The amplitude of the traveling wave on the elastic wall is so large that at the narrowest point the wall pressed by each other.Peristaltic transport of two layered viscous incompressible fluid is studied theoretically through the axisymmetric geometry. An attempt has been made to examine the rate of positive pumping against the decrease of friction force at the innerwall of the tube.

Over the past few years, analytical and experimental studies have been carried out to analyze the flow parameters under the peristaltic transport. Burns et. al. [1] studied the peristaltic motion through a pipe under the assumptions of small Reynold's number. Fung et. al. [2] analyzed the flow of urine with peristaltic action in a two-dimensional channel. Shapiro et. al. [3] analyzed the peristaltic pumping using long wave length at low Reynold's number for mechanical effectiveness in relevance to ureter function. Weinberg et. al. [4] made the experimental investigations on two-dimensional peristaltic pumping to measure the flow parameters at fixed locations of the tube. Tin-Kanet. al. [5] analyzed the insight into the mechanism of solid particle by peristaltic. Shuklaet. al. [6] observed the peristaltic transport of a power-law fluid with variable consistency. Srivastava et. al. [7] described the peristaltic transport of blood using Casson model under zero Reynold's number. Liepsch [8] discussed a detailed discussion on blood circulatory system by which the human heart operates as a double working pump for the flow of blood similar to a piston in the tube network. He also described about the contraction and expansion waves produced by the pumping action using Newtonian and non-Newtonian nature of blood. Usha et al. [9] investigated the effects of curvature and inertia on the peristaltic transport in two fluid system. Basavarajappaet. al. [10] described the peristaltic transport of two-layered viscous incompressible fluid to approximate the stream functions and the interface. Vajraveluet. al. [11] analyzed the peristaltic transport of a Herschel-Bulkley fluid in contact with Newtonian fluid. G. Radhakrishnamacharya, Ch. Srinivasulu[12] analyzed the effects of pertinent parameters on temperature and heat transfer.DulalChandra Sanyal, Ananda Biswas [13] were discussed by assuming blood to be incompressible viscous Newtonian fluid. EthemToklu [14] developed a new mathematical model for peristaltic transport in the esophagus which is monomeric measurements of luminal pressure have been obtained in the esophagus and interpreted both biological and mechanical point of view. AbdelhalimEbaid et.al. [15] explained the influence of viscosity variation on peristaltic flow in an asymmetric channel in view of new exact solutions. These solutions are used to study the effects of viscosity parameter, Daray's number, porosity, amplitude ratio, Jeffrey fluid parameter and amplitudes of the waves on the pressure rise and the axial velocity. B J Gireeshaet.al. [16] investigated non linear radiative Casson –Carreau liquid models considering the aspects of homogeneous and heterogeneous reactions and have shown that the liquid velocities in case of Casson fluid is higher than the carreau fluid with varying magnetic parameter. A Tanveer et al [17] investigated the effects of slip condition and joule heating on peristaltic flow of Bingham nano fluid and presented the formulation under the assumption of long wavelength and small Reynolds number.

In view of the research done by various authors, it is necessary to understand the effect of varying wall moment on the peristaltic transport with a reference to Herschel-Bulkley fluid flow to compare the physiological flow parameters. Newton-Raphson method is used to determine the interface between core layer and peripheral layer employing the iterations with six stages in each approximation.

2. FORMULATION

Consider the peristaltic transport of axi-symmetric flow of Herschel-Bulkley fluid through the tube of radius 'a' of which the core layer with radius h_1 is filled with 90 % of the Herschel-Bulkley fluid with another immiscible fluid as plasma in peripheral layer with radius as h. λ is the wavelength when the wall is subjected to periodic peristaltic movement. The flow becomes steady in the reference frame moving in the direction of the wave propagation with speed 'c'. Geometry of the fluid flow is sinusoidal, travelling with amplitude b_1 and b in two layers respectively.

Under the peristaltic action R = H(z) be the instantaneous radius when the fluid is surrounded co axially. (R , Z) be the fixed frame and (R', Z') be the reference frame, then $z' = Z + ct', r' = R, w_i' = w_i + c$

Taking Herschel-Bulkley fluid, the non-Newtonian fluid is modeled as,

$$\tau = \begin{cases} \mu\, e^{n_i} + \tau_0 & \tau \geq \tau_0 \\ e = 0 & \tau < \tau_0 \end{cases} \quad (1)$$

Where τ- shear stress,τ_0- yield stress, $e = \dot{\gamma}$– deformation rate,μ - viscosity and n_i- fluid behavior index.
$i = 1$ (core layer), $i = 2$ (peripheral layer),
μ_1- viscosity of chime layer, μ_2- viscosity of mucus layer,
$m_r = \mu_r = \mu = \frac{\mu_1}{\mu_2} in h_1 \leq r \leq h$
$m_r = \mu_r = \mu = 1 in 0 \leq r \leq h$

Geometry of the tube wall is taken for cylindrical polar coordinates (r, θ, z) to study the problem,

$$H(z) = a + b\sin\left(\frac{2\pi z}{\lambda}\right)$$

$$H(z) = 1 + \epsilon \sin\left(\frac{2\pi z}{\lambda}\right) \quad (2)$$

Where $\epsilon = \frac{b}{a}$ the amplitude ratio
For varying wall thickness, we consider
$h(\eta) = a + \eta \cos(c - \lambda z)$
H(z) is taken as $h(\eta)$.
Where a =tube radius, η = amplitude ratio, c = wave speed, λ = wave length,
z = axial coordinate

Using long wave length approximation, neglecting wall slope and inertia forces for steady flow under lubrication theory, the equations of motion is

$$\frac{\partial p'}{\partial z'} = \frac{1}{r'}\frac{\partial}{\partial r'}\left\{r' m_r \frac{\partial w_i'}{\partial r'}\left|\frac{\partial w_i'}{\partial r'}\right|^{n_i-1}\right\} \quad (3)$$

$$\frac{\partial p'}{\partial z'} = 0 \quad (4)$$

Non-dimensional quantities are,

$$r = \frac{r'}{a} \quad z = \frac{z'}{\lambda} \quad h_1 = \frac{H_1}{a} w_i = \frac{w_i'}{a} m_r = \frac{\mu_2}{\mu_1} \quad p = \frac{p' a^{n_i+1}}{m_1 \lambda c^{n_i}} \quad (5)$$

3. ANALYSIS

Equation (3) becomes

$$\frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left\{r\, m_r \frac{\partial w_i}{\partial r}\left|\frac{\partial w_i}{\partial r}\right|^{n_i-1}\right\}$$

$$\frac{\partial p}{\partial z} = p \quad \text{(constant)}$$

$$p = \frac{1}{r}\frac{\partial}{\partial r}\left\{r\, m_r \frac{\partial w_i}{\partial r}\left|\frac{\partial w_i}{\partial r}\right|^{n_i-1}\right\}$$

Equation (7) becomes

$$\left\{\frac{\partial w_i}{\partial r}\left|\frac{\partial w_i}{\partial r}\right|^{n_i-1}\right\} = \frac{pr}{2m_r}$$

The conditions at the fluid interface are the continuity of the velocity and the stress across it. Then

$$\left|\frac{\partial w_i}{\partial r}\right| = \left|\frac{pr}{2m_r}\right|^{\frac{1}{n_i}} \quad (6)$$

For the binary system we consider$\varphi^c + \varphi^p = 1$.
The conservation of mass for each phase can be taken to be as,

$$\frac{\partial \varphi^c}{\partial t} + \nabla.\left(\varphi^\beta v^\beta\right) = 0$$

where the velocity $v^c = \frac{\partial w_i}{\partial t}$
The overall continuity equation is obtained by adding the equations for both phase so that,

$$\nabla.(\varphi^c v^c + \varphi^p v^p) = 0 \quad (7)$$

$$\nabla.v^f = 0 \quad (8)$$

We define $v^f = (\varphi^c v^c + \varphi^p v^p)$ as a macroscopic fluid vector. The momentum equation for each phase can be modeled as,

$$\rho^\beta\left[\frac{\partial v^\beta}{\partial t} + \left(v^\beta.\nabla\right)v^\beta\right] = \nabla.T^\beta + \rho^\beta b^\beta + \pi^\beta \quad (9)$$

where T^β is the stress tensor and for the β phase, b^β is the resultant external body force (neglected here) and π^β is a drag force between the constituents representing inertial forces due to frictional interaction between the two layers. For small velocities and deformation rates, the inertia terms can be assumed to be negligible. With these assumptions equation (9) becomes

$$\nabla.T^\beta = -\pi^\beta \quad (10)$$

Newton's third law implies $\pi^c = -\pi^p$. Then the stress tensor can be modeled as

$$T^\beta = -\varphi^\beta PI + \sigma^\beta \quad (11)$$

$$-\pi^c = \pi^p K(v^c - v^p) - P\nabla\varphi^s \quad (12)$$

Where σ^p represents a stress, 'K' is the drag coefficient of relative motion; 'P' is the pressure and 'I' is the identity tensor. These stress equations split the stress tensors into contributions due to hydrostatic pressure and those due to viscous stress. The interaction term represents the linear drag between the constituents.

Substituting the interaction terms of equation (12) in to equation (10) and using

$\sigma^p = 1 - \varphi^c$ leads to

$$\nabla.\sigma = \frac{k}{\varphi^f}(v^c - v^p)$$

We get,

$$v^f = -1 - |C_2|^{k_2}\frac{[a+\eta\cos(c-\lambda z)]^{mk_2+1}}{mk_2+1} + |C_2|^{k_i}\frac{r^{mk_i+1}}{mk_i+1} \quad (13)$$

For the peristaltic transport of two layered Herschel-Bulkley fluid in circular tube in an axi-symmetric flow $0 \le r \le h_1$ and $h_1 \le r \le h$

$$v^f = [[C_2 r^{m-1}]]^{\frac{1}{n_i}}\left\{\frac{(R-R_p)^{k_i+1} - (r-r_p)^{k_i+1}}{k_i+1}\right\}$$

For the core layer r - h_1 and for the mucus layer $h_1 - h$

$$v_i^f = -1 - |C_2|^{k_i}\left[\frac{r^{mk_i+1} - h_1^{mk_i+1}}{mk_i+1}\right] + |C_2|^{k_2}\left[\frac{h_1^{mk_2+1} - [a+\eta\cos(c-\lambda z)]^{mk_2+1}}{mk_2+1}\right] \quad (14)$$

The flow rate q_1: $0 \le r \le h_1$ [For chyme layer]

$$q_1 = 2\int_0^{h_1} r v_1^f \, dr$$

$$q_1 = -h_1^2 - |C_2|^{k_1}\left[\frac{h_1^{mk_1+3}}{mk_1+3}\right] + |C_2|^{k_2}\left[\frac{h_1^{mk_2+3} - [a+\eta\cos(c-\lambda z)]^{mk_2+1} h_1^2}{mk_2+1}\right] \quad (15)$$

The flow rate q_2: $h_1 \le r \le h$ [For mucus layer]

Where $h(\eta) = a + \eta\cos(c - \lambda z)$

$$q_2 = 2\int_{h_1}^{h} r v_2^f \, dr$$

$$q_2 = -[a+\eta\cos(c-\lambda z)]^2 + h_1^2 - |C_2|^{k_2} \times$$

$$\times\left\{\begin{array}{c}\dfrac{-[a+\eta\cos(c-\lambda z)]^{mk_2+3}}{mk_2+3} + \\ + \dfrac{h_1^2}{(mk_2+1)(mk_2+3)}\left[2h_1^{mk_2+1} - (mk_2+3)[a+\eta\cos(c-\lambda z)]^{mk_2+1}\right]\end{array}\right\} \quad (16)$$

The flow rate 'q' across any cross section is independent of 'z' under lubrication approach. Then the instantaneous volume flow rate in terms of two layers is given by

$$q = q_1 + q_2$$

$$q = -[a+\eta\cos(c-\lambda z)]^2 - |C_2|^{k_1}\frac{h_1^{mk_1+3}}{mk_1+3} + |C_2|^{k_2}\left[\frac{h_1^{mk_2+3}}{mk_2+3} - \frac{[a+\eta\cos(c-\lambda z)]^{mk_2+3}}{mk_2+3}\right] \quad (17)$$

Dimensionless time average flux in terms of flow rate is obtained as,

$$Q = \left\{\begin{array}{c}-[a+\eta\cos(c-\lambda z)]^2 - |C_2|^{k_1}\dfrac{h_1^{mk_1+3}}{mk_1+3} + \\ + |C_2|^{k_2}\left[\dfrac{h_1^{mk_2+3}}{mk_2+3} - \dfrac{[a+\eta\cos(c-\lambda z)]^{mk_2+3}}{mk_2+3}\right]\end{array}\right\} + 1 + \frac{\epsilon^2}{2}$$

Where ϵ- the amplitude ratio

The stream functions ψ_1 and ψ_2 in terms of velocities are given by,

$$v_1^f = -\frac{1}{r}\frac{\partial\Psi_1}{\partial r} \quad \text{or} \quad \frac{\partial\Psi_1}{\partial r} = -r\,v_1^f$$

$$\Psi_1 = \frac{r^2}{2}\left[1 - |C_2|^{k_1}\left\{\frac{2r^{mk_1+1} - (mk_1+3)h_1^{mk_1+1}}{(mk_1+3)(mk_1+1)}\right\} - |C_2|^{k_2}\left\{\frac{h_1^{mk_2+1} - [a+\eta\cos(c-\lambda z)]^{mk_2+1}}{(mk_2+1)}\right\}\right] \quad (18)$$

$$v_2^f = -\frac{1}{r}\frac{\partial\Psi_2}{\partial r} \text{or} \frac{\partial\Psi_2}{\partial r} = -r\,v_2^f$$

$$\Psi_2 = $$

$$= \left[\left[\frac{2q + r - [a+\eta\cos(c-\lambda z)]^2}{2}\right] - \left\{\frac{2r^{mk_2+3} - (mk_2+3)r^2[a+\eta\cos(c-\lambda z)]^{mk_2+1} + (mk_2+1)[a+\eta\cos(c-\lambda z)]^{mk_2+1}}{2(mk_2+3)(mk_2+1)}\right\}\right] \quad (19)$$

Taking $\Psi_1 = \frac{q_1}{2}$ or $\Psi_1 = \frac{q_1}{2}$
For stream functions at $k_1 = k_2 = k$ we get,

$$\Psi_1 = \frac{r^2}{2} + \frac{q+h^2}{h^{mk+3}(mk+1)}\left[r^{mk+3} - (mk+3)h_1^{mk+1}r^2 + (mk+3)[a+\eta\cos(c-\lambda z)]^{mk+1}\frac{r^2}{2}\right] \quad (20)$$

$$\Psi_2 = \left[\frac{2q+r^2+[a+\eta\cos(c-\lambda z)]^2}{2}\right] + \\ + \frac{q+h^2}{2h^{mk+3}(mk+1)}\left\{\begin{matrix} 2r^{mk+3} - (mk+3)[a+\eta\cos(c-\lambda z)]^{mk+1}r^2 \\ . \\ +(mk+1)[a+\eta\cos(c-\lambda z)]^{mk+3}\end{matrix}\right\} \quad (21)$$

For $h_1 = 0$

$$L(0) = [a+\eta\cos(c-\lambda z)]^{mk+3}(mk+1)\{q_1 - q - [a+\eta\cos(c-\lambda z)]^2\} - \\ - \{q + [a+\eta\cos(c-\lambda z)]^2\}(mk+1)[a+\eta\cos(c-\lambda z)]^{mk+3} \quad (22)$$

For $h_1 = h$

$$L[a+\eta\cos(c-\lambda z)] = [a+\eta\cos(c-\lambda z)]^{mk+3}(mk+1) \\ \{q_1 - q - [a+\eta\cos(c-\lambda z)]^2\} \quad (23)$$

Equation of the interface is obtained for a particular case at $h_1 = \alpha$ and $\frac{1}{n_1} = \frac{1}{n_2}$ with the condition that the flow rate in core layer is twice the stream function ($r = h_1$). Substituting equation (17) in q_1 and further $Q_1 = q_1 + h^2$ gives

$$Q_1 = 2\left\{\frac{\alpha^5 - \alpha^3\{[a+\eta\cos(c-\lambda z)]^{mk+3} + 7\bar{Q}\} - \alpha\{[a+\eta\cos(c-\lambda z)]^4 + [a+\eta\cos(c-\lambda z)]^2 - 16\bar{Q}\alpha^2\}}{-13\alpha^4 - 3\alpha^2[a+\eta\cos(c-\lambda z)]^2 - 23\alpha^2}\right\} \quad (24)$$

For a given flow rate with non uniform interface ($Q_1 = 0$), equation (24) is reduced to 5th degree polynomial in ‘ α ’. Newton-Raphson method is employed with ten iterations in each step to obtain the set of values for α . Series of values of α represent interface. The initialization is made with α = 0.3 to 2.6143 chosen as sufficiently close to the root in comparison with H(z).

The pressure gradient under the assumptions of (24) with $\frac{1}{n_1} = \frac{1}{n_2} =$ nis obtainedby separating P from equation (6) and for single layer model. m_r= 1 and n =1.

For various amplitudes, fifth degree equations are proposed and solved numerically as,

$$\alpha = 0, \text{ and } \alpha^4 + 16.25\alpha^3 - \{[a+\eta\cos(c-\lambda z)]^2 + 7.7\}\alpha^2 + \\ -\{[a+\eta\cos(c-\lambda z)]^4 + [a+\eta\cos(c-\lambda z)]^2\} = 0 \quad (25)$$

Here we claim that numerically $\alpha \neq 0$, since the radius of the artery ranges from 0.3mm to 2.6143mm
Again by equation (25),

$$[a+\eta\cos(c-\lambda z)]^4 + (\alpha^2 - 3.75\alpha + 1)[a+\eta\cos(c-\lambda z)]^2 + \\ +(-\alpha^4 - 16.25\alpha^3 + 7.7\alpha^2 - 11.15\alpha) = 0 \quad (26)$$

Tube radius- a=1, Wave speed- c=6, Wave length-λ=9, $\bar{Q} = 1.1$ are substituted to equation (26) and
For $\eta = 0.2, 0.4, 0.6$ and 0.8 , we obtain the corresponding values of $h(\eta) = \alpha$ as, α: ranging from 0 to 0.078125
The non-uniform wall moment variation (Q_1) is approximated by using Halving method.
The expressions for pressure difference (Δp) studied between the extreme locations of each wavelength as,

$$\Delta p = \frac{-8\left\{[a+\eta\cos(c-\lambda z)]^2 - \frac{C_2^k h_1{}^{mk_1+3}}{(mk_2+3)} + C_2^k\left\{\frac{h_1^{mk_2+3}}{(mk_2+3)} - \frac{[a+\eta\cos(c-\lambda z)]^{mk_2+3}}{(mk_2+3)}\right\}\right\}[2+3\epsilon^2]}{[1-\epsilon^2]^{7/2}} - \frac{8}{[1-\epsilon^2]^{3/2}} \quad (27)$$

Friction force at the wall is given by,

$$F = \frac{8\left\{[a+\eta\cos(c-\lambda z)]^2 - \frac{C_2^k h_1{}^{mk_1+3}}{(mk_2+3)} + C_2^k\left\{\frac{h_1^{mk_2+3}}{(mk_2+3)} - \frac{[a+\eta\cos(c-\lambda z)]^{mk_2+3}}{(mk_2+3)}\right\}\right\}}{[1-\epsilon^2]^{3/8}} + 8$$

$$F = \frac{8}{[1-\epsilon^2]^{3/8}} -$$

$$-[a+\eta\cos(c-\lambda z)]^2 - \left[\frac{p}{2m_r}\right]^{1/n_1}\left[\frac{h_1^{\frac{1}{n_1}+3}}{\frac{1}{n_1}+3}\right] - \left[\frac{p}{2m_r}\right]^{1/n_2}\left\{\frac{h_1^2[a+\eta\cos(c-\lambda z)]^{\frac{1}{n_2}+3} - h_1^{\frac{1}{n_2}+3}}{\frac{1}{n_2}+1}\right\} +$$

$$+\left[\frac{p}{2m_r}\right]^{1/n_2}\left\{\frac{-[a+\eta\cos(c-\lambda z)]^{\frac{1}{n_2}+3}}{\frac{1}{n_2}+3} + \frac{\left(\frac{1}{n_2}+3\right)h_1^2[a+\eta\cos(c-\lambda z)]^{\frac{1}{n_1}+1} - 2h_1^{\frac{1}{n_2}+3}}{\left(\frac{1}{n_2}+1\right)\left(\frac{1}{n_2}+3\right)}\right\} \qquad (28)$$

From finite range of flux $\overline{Q}_L$ the limitation of the flux $\overline{Q}_L$ is calculated as

$$\overline{Q}_L = \frac{\varepsilon^2}{2} + \left[\frac{\alpha^{k+3} - (k+1) + 2(k+3)\alpha^{k+2} - (k+3)\alpha^{k+2} - (k+3)\alpha^{k+3}}{4\alpha^{k+1} - 2(k+3)\alpha^{k} + (k+3)\alpha^{k+1} - (\alpha+3)}\right] \qquad (29)$$

The ratio of average pressure rise and the time averaged flux is determined as $R_{f.}$,

$$R_f = \left[\frac{-8\left\{[a+\eta\cos(c-\lambda z)]^2 - \frac{C_2^k h_1^{mk_1+3}}{(mk_2+3)} + C_2^k\left\{\frac{h_1^{mk_2+3}}{(mk_2+3)} - \frac{[a+\eta\cos(c-\lambda z)]^{mk_2+3}}{(mk_2+3)}\right\}\right\}(2+3\epsilon^2)}{(1-\epsilon^2)^{7/2}\left\{[a+\eta\cos(c-\lambda z)]^2 - \frac{C_2^k h_1^{mk_1+3}}{(mk_2+3)} + C_2^k\left\{\frac{h_1^{mk_2+3}}{(mk_2+3)} - \frac{[a+\eta\cos(c-\lambda z)]^{mk_2+3}}{(mk_2+3)}\right\}+1+\frac{\epsilon^2}{2}\right\}} - \frac{8}{(1-\epsilon^2)^{7/2}\left\{[a+\eta\cos(c-\lambda z)]^2 - \frac{C_2^k h_1^{mk_1+3}}{(mk_2+3)} + C_2^k\left\{\frac{h_1^{mk_2+3}}{(mk_2+3)} - \frac{[a+\eta\cos(c-\lambda z)]^{mk_2+3}}{(mk_2+3)}\right\}+1+\frac{\epsilon^2}{2}\right\}}\right] \qquad (30)$$

4. RESULT AND DISCUSSION

Due to wall varying in the axial direction the effect of peristaltic transport gives the importance of radial velocity and its effect on axial velocity distribution. The instantaneous velocity gets disturbed due to varying wall moment. Velocities above the centerline are radial in the irrotational motion with mucus in the core layer and chyme in the peripheral layer. The positive pumping is derived at non-varying wall and the change in the heights of sinusoidal waves is appeared in the varying wall. The slope of the wall decreases and causes the disturbance to deviate from a parabolic slope.

Due to varying temperature the velocity profiles are stagnant. This is shown in graph (fig.5) for the numerical computation of fifth degree equations for $\alpha = f(h)$, at ϵ =0.2, 0.4, 0.6, 0.8 to achieve the positive pumping. Frictional force gives the influence of varying wall moment on peristaltic flow in the axial direction.

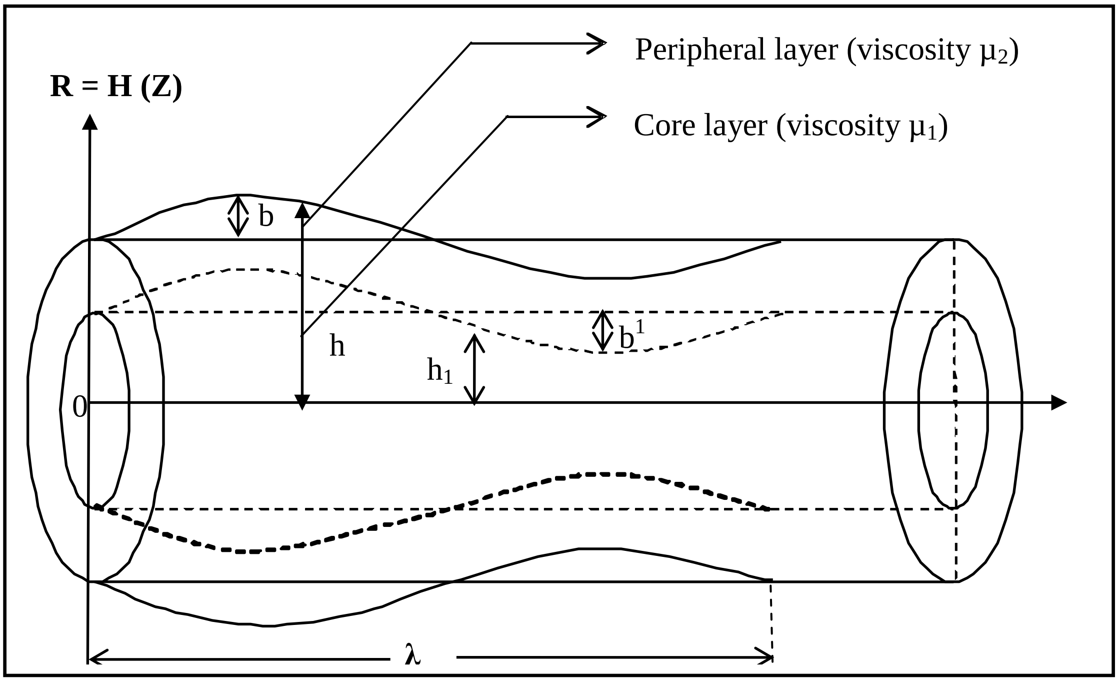

Figure 1: Geometry of two layered Peristaltic Transport

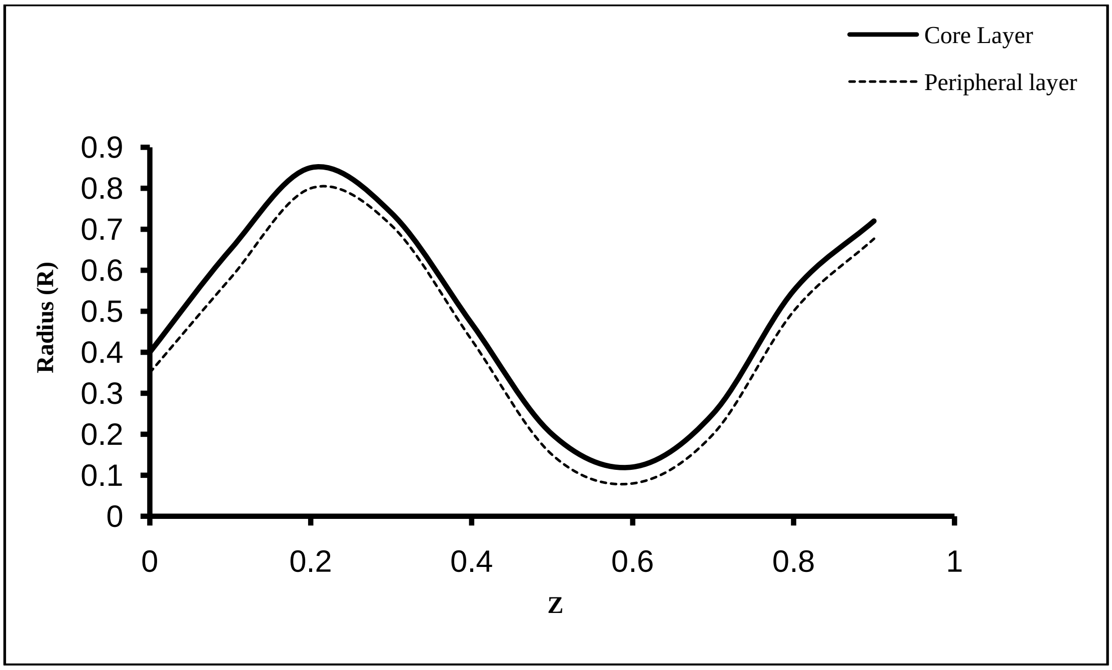

Figure 2: Peristaltic Transport in Axi-symmetric Tube

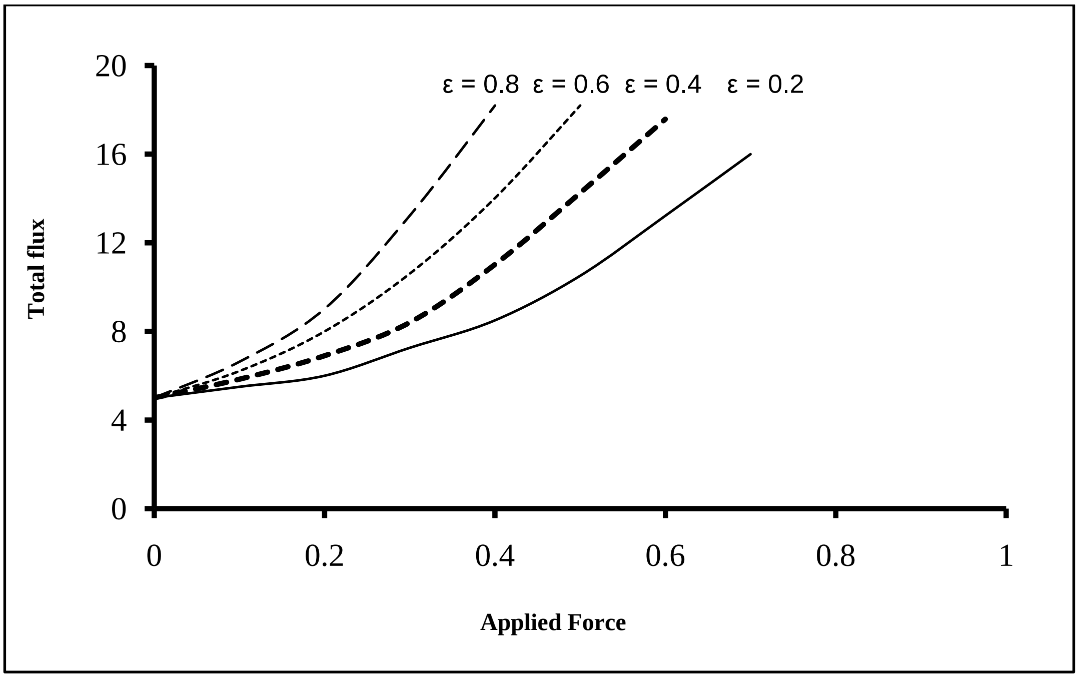

Figure 3: Total Flux (Q) Vs Applied force (F)

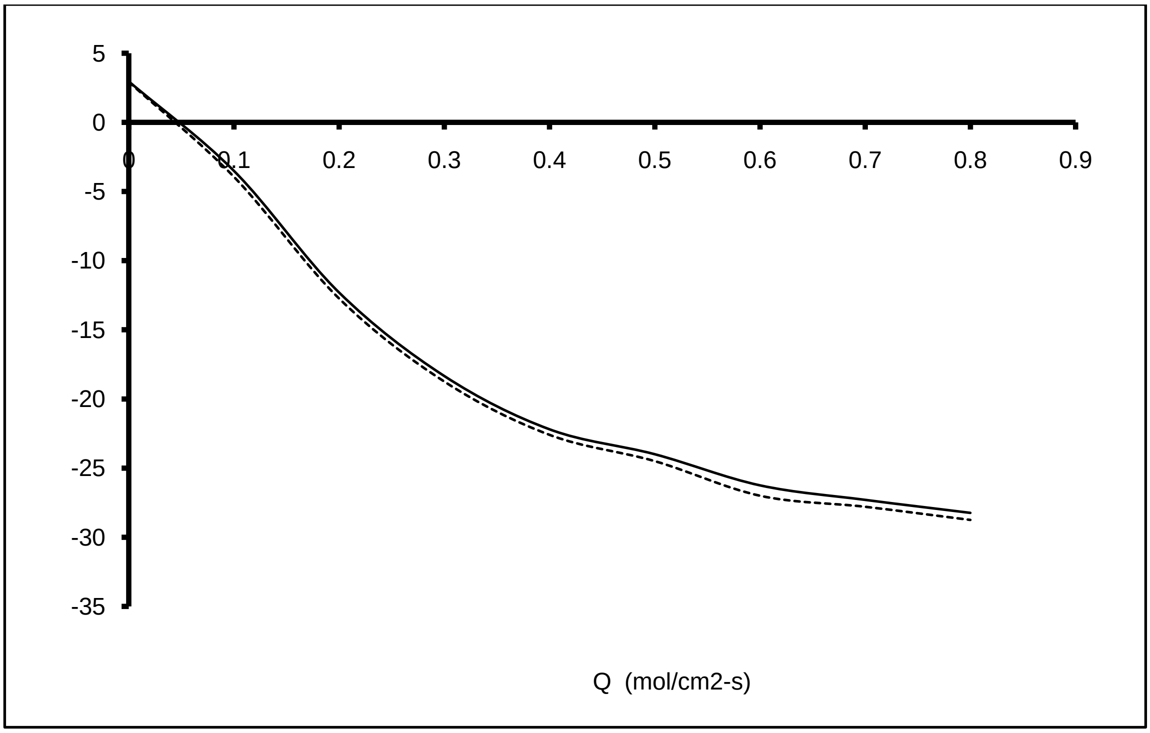

Figure 4: Variation of Force (F) v/sFlux(Q)

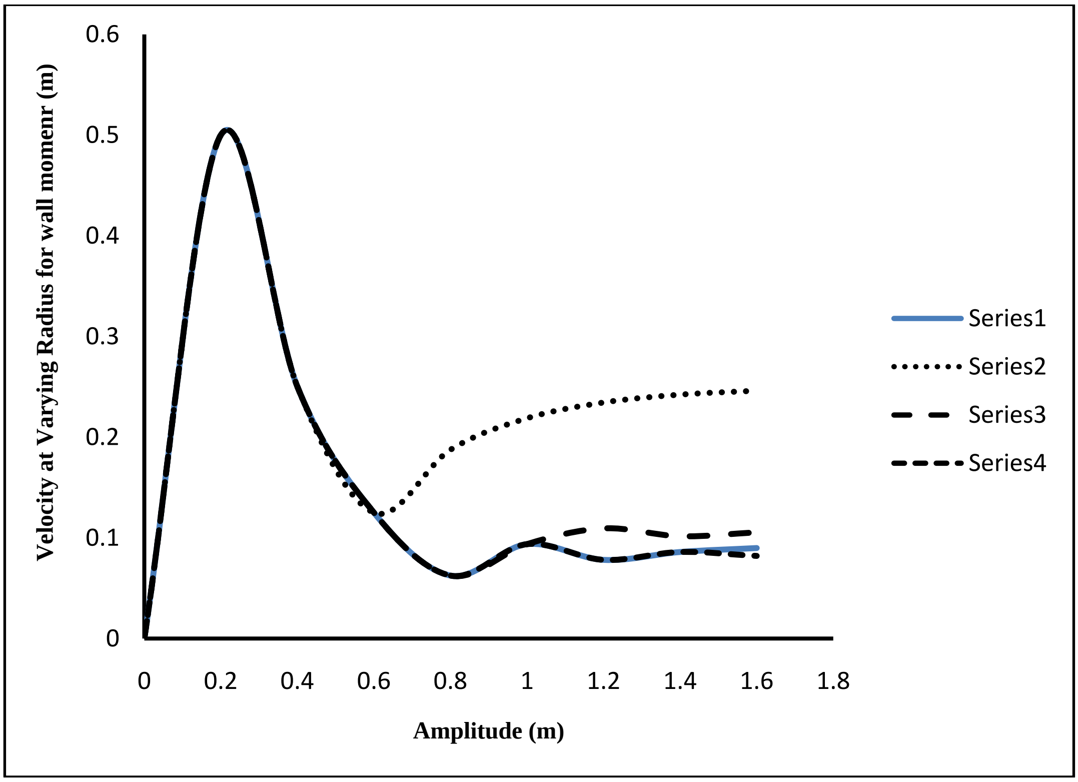

Figure 5: Varying wall moment

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